R A N - 2 2 0 3 0 0 0 2 0 5 0 2 3 0 0 2

RAN-2203000205023002

T. Y. B. Sc. (Sem.-V) Examination October - 2025
Mathematics (MTH-502)Linear Algebra-I (OLD)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
(2) All questions are compulsory.
(3) Figures to the right indicate marks of the questions.
(4) Follow usual notations.

Seat No.:

Student's Signature

Q.1. Answer the following (Any Five): **(10)**

(1) Is $f : (x.y) \to |x - y|$ a binary operation on Z ? Justify your answer.
(2) In a vector space R^+,for every $u \in R^+$ and any scalar α, if $\alpha \cdot u = u^\alpha$ then find Identity and Inverse element with respect to the operation multiplication.
(3) Is $M= \{(x_1,x_2) \in V_2/x_1^{\,2} = 0)\}$ be a subspace of V_2 ? Justify your answer.
(4) Find the span of Y –axis and XZ – plane in vector space V_3.
(5) For what values of α_v for $- v_1 + v_2 - v_3 + \cdots + \alpha_i v_i + \cdots + 0v_n$ be a Non trivial linear combination?
(6) Let $U= yz$– plane *and* $W = z$ – axies . Is $Z = U \oplus W$? Justify your answer.
(7) Is a basis can never include the zero vector ? Justify your answer.
(8) Write standard basis of a vector spaces V_4 and V_5.

Q.2. Answer the following (Any two): **(10)**

(1) Prove that the set $\{(x,2x, - 5x,x) \in V_4 / x \in R\}$ is a commutative group with respect to coordinate wise addition.
(2) Prove that the set $\{\alpha_1 u_1 + \alpha_1 u_2 + --- + \alpha_n u_n / \alpha_i \in R, u_i \in V, \forall i = 1 \text{ to } n \}$ is a subspace of vector space V.
(3) Let U and W be two subspace of vector space V then prove that $(U \cap W)$ be a subspace of V.

Q.3. Answer the following (Any two): **(10)**

(1) If U and W are two subspaces of a vector space V then prove that $U + W$ is a subspace of V with condition that $U + W = [U \cup W]$.
(2) Determine which of the following vector belongs to the span of a set
$S = \{(1,2,1), (1,1, - 1), (4,5, - 2)\}$ (*a*) (1,-3,5) (*b*) $(\frac{-1}{3}, \frac{-1}{3}, \frac{1}{3})$
(3) Let $S = \{v_1, v_2, ... ,v_n\}$ be a non empty subset of a vector space V then prove that (*a*) $[v_1, v_2, v_2, ... ,v_n] = [1v_1,2v_2 ,3v_2, ..., nv_n]$.
(b) $[v_1, v_2] = [v_1 - v_2, v_1 + v_2]$

Q.4. Answer the following (Any two): **(10)**

(1) In a vector space V, prove that
(a) If the set $\{v_1,v_2, ..., v_n\}$ is L,I and $v \in [v_1,v_2, ..., v_n]$ in a vector space V then the set $\{v,v_1,v_2, ... ,v_n\}$ is L.D.

(b) If the set $S = \{v_1, v_2, ... ,v_n\}$ is L,I and $v \notin [S]$ in a vector space V then the set $\{v, v_1, v_2, ... ,v_n\}$ is L.I.

(2) If u, v and w are linearly independent vectors then prove that the set $\{u + v - w, u + v - w, u + v + w\}$ is also L.I. Are the vectors $u + v, v + w, w + u$ L.I ? Justify your answer.

(3) If the set $S_4 = \{(1,1,0), (0,1,1), (1,1,-1), (1,1,1)\}$ is L.D then find largest L.I subset S_3 of S_4 with condition $[S_3] = [S_4]$.

Q.5. Answer the following (Any two): (10)

(1) If a vector space V has a basis B_1 containing k elements then prove that every other basis B_2 has also k elements.

(2) (a) If V has a basis of n elements then prove that every set of p vectors with $p > n$ is L.D.

(b) Is the set $\{(2,2,2), (5,-5,-5), (3,-7,-7)\}$ a basis of a vector space V_3? Justify your answer.

(3) Find the general form of a co-ordinate vector of a vector (1,2,1) relative to a basis $\{(2,1,0), (2,1,1), (2,2,1)\}$.